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T-13: Abstractions

Task objectives vary from concrete to abstract over a wide range. Rinsing paintbrushes and making sandwiches are concrete tasks. Modeling the structure of organizations and comparing literary periods are abstract tasks. Concrete objectives involve the physical manipulation of tangible objects whereas abstract objectives involve the mental manipulation of intangible objects. The development of this knowledge construction function orients students to look for the possibilities that may exist to secure easier ways to solve problems either by raising or lowering their abstractness.

Prior to the development of this knowledge construction function students often miss opportunities to modify abstractness, up or down, as an aid to problem solving. The result is that tasks that might have been solved with the help of this knowledge construction function instead are botched or abandoned.

Remember the saying that someone cannot see the forest for all the trees? To mediate the function of abstractions have your students practice switching their focus between the trees and the forest across many different subject areas. Have them practice going from concrete cases to abstract rules or principles and from abstract rules or principles to real and concrete cases. The abstraction can be in any subject area, e.g. a rule of grammar (e.g. rules of punctuation), a rule of science (e.g. gravity), a rule of law (e.g. stealing) or a rule in geometry (Pythagorean Theorem).

Have students create examples ("instances") of each rule and have them learn to go back and forth: From abstract rules to concrete instances; from concrete instances to abstract rules. For example, have them discuss the concept of gravity and have them experience concrete examples of it: "Open your hand and drop an eraser". Watch a video clip of objects floating in the Space Station. Discuss the abstract principle and use the concrete examples as illustrations. - Have them take the concrete example of a burglary and have them discuss the rules of law under which the convicted burglar was sentenced. Help your students discover how they can use this knowledge construction function to search for abstract rules or principles when given concrete situations and search for concrete examples when given abstract rules or principles.

The development of abstractions enables the learner to comprehend the particular in light of the general and to illustrate the general with the help of the particular. The development of this knowledge construction function is often accompanied by a growing propensity in the learner to search for commonalities among his discrete experiences. These represent increases in abstraction. This in turn produces a need for new concepts, verbal tools and grammar to articulate the newly discovered superordinate aspects of experience. Statements modified by `all', `none', `some', `always', `never', `most of the time' and `some of the time' all indicate attempts by the learner to mark out identified commonalities among his various experiences.

The abstraction of cognitive categories is strongly supported by the acquisition of language and is promoted by the modeling and feedback that teachers provide through their interactions with the child. For example, the abstract concept of fruit may be provided together with the concrete instances of apple and orange. Once acquired, abstract cognitive categories permit both distinctions and generalizations to be made which previously might have escaped the learner. Note also that the abstraction of cognitive categories as a knowledge construction function is a precursor to rule seeking and the establishment of cause-effect relationships (T-25).

Have your students discuss different strategies to lower or to raise abstractness in order to better solve problems. For example, creating models, blueprints or maps lowers abstractness by providing a concrete representation of the relationships under consideration. Creating mathematical formulas raises abstractness by providing a rule-based representation of concrete situations.

The nemesis of many students is the infamous word problem. It provides a good illustration of the need to mediate the development of the knowledge construction function of abstractions. Below are three detailed examples of word problems. In each case the verbal information can be rewritten as simple algebraic equations that are readily solved. The difficulties students have with word problems are not in solving the mathematical statements. It is with the change in abstraction that is required to generate the mathematical statements to begin with. Here are three typical word problems:

#1: Sixty-three more than 8 times a number is 145. What is the number?

#2: One number is thirty-six more than five times another number. If each number were multiplied by 6, their difference would be 264. What are the numbers?

#3: A barge has speed over water of 6 miles per hour. A river flows downstream at the speed of 2 miles per hour. How long will it take the barge to go from point A to point B upstream, and then back, if the distance from A to B is 32 miles?

Let's look briefly at each problem in turn.

Problem #1: Sixty-three more than 8 times a number is 143. More abstractly:

$8x + 63$	$= 143$	$\Leftrightarrow$
$8x$	$= 143 - 63$	$\Leftrightarrow$
$8x$	$= 80$	$\Leftrightarrow$
x	$= 10$	

Problem #2: One number is thirty-six more than five times another number. If each number were multiplied by 6, the difference between them would be 264. What are the numbers?" More abstractly:

y	$= 5x + 36$	
$6y - 6x$	$= 264$	
$6(5x + 36) - 6x$	$= 264$	$\Leftrightarrow$
$30x + 216 - 6x$	$= 264$	$\Leftrightarrow$
$24x$	$= 48$	$\Leftrightarrow$
x	$= 2$	
y	$= 10 + 36$	$\Leftrightarrow$
y	$= 46$	

Problem #3. A barge has speed over water of 6 miles per hour. A river flows downstream at the speed of 2 miles per hour. How long will it take the barge to go from point A to point B upstream, and then back, if the distance from A to B is 32 miles?" More abstractly:

Speed Up	= 6 – 2 miles per hour	= 4 miles per hour

Speed Down	= 6 + 2 miles per hour	= 8 miles per hour
Time Up	= 32 miles/4 miles per hour	= 8 hours
Time Down	= 32 miles/8 miles per hour	= 4 hours
Total Time	= 8 + 4 hours	= 12 hours

Students with this knowledge construction function tend to move more readily between concrete and abstract versions of information. The function helps them both to increase the abstractness, as in the above cases, and to decrease abstractness as when creating additional examples of the same type of problem.

It is important to maintain a distinction between complexity and abstractness. The complexity of a task objective is determined by the number of pieces of information that have to be processed. The higher the number the greater the complexity. The abstractness of a task objective is determined by the concrete vs. intangible nature of the pieces of information that have to be processed. The more intangible, the higher the abstractness. Finding a needle in a haystack is a very complex task but not a very abstract one. The understanding that a small amount of mass may be converted into a large amount of energy, and vice versa, as expressed in the special theory of relativity, is a very abstract one but, as can be seen from the equation $E = mc^2$ not a very complex one: Energy is equal to mass multiplied by the square of the velocity of light. Complexity and abstractness cannot be measured precisely. They are task characteristics that interact with the perception of the person confronted with the task. What is a complex or an abstract task to one learner may be a simple or concrete task for another. The perceptions of both complexity and abstractness also change within learners as they acquire greater proficiency and expertise. What used to be hard is subsequently seen as easy.